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LXX. On the number of corpuscles in an atom

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LXX. *On the Number of Corpuscles in an Atom.* *By* Prof. J. J. Thomson, *M.A.*, *F.R.S.**

I CONSIDER in this paper three methods of determining the number of corpuscles in an atom of an elementary substance, all of which lead to the conclusion that this number is of the same order as the atomic weight of the substance. Two of these methods show in addition that the ratio of the number of corpuscles in the atom to the atomic weight of the element is the same for all elements. The data at present available indicate that the number of corpuscles in the atom is equal to the atomic weight. As, however, the evidence is rather indirect and the data are not very numerous, further investigation is necessary before we can be sure of this equality; the evidence at present available seems, however, sufficient to establish the conclusion that the number of corpuscles is not greatly different from the atomic weight.

It will be seen that the methods are very different and deal with widely separated physical phenomena; and although no one of the methods can, I think, be regarded as quite conclusive by itself, the evidence becomes very strong when we find that such different methods lead to practically identical results.

To enable the argument to be more easily followed, I shall begin with a general description of the methods and the results to which they lead, and postpone the details of the theory of each of the methods to the latter part of the paper.

The first method is founded on the dispersion of light by gases. If we regard an atom as consisting of a number of corpuscles dispersed through a sphere of uniform positive electrification, it is evident that the dispersive power of a medium consisting of these atoms will depend upon the mass of the positive electrification as well as upon the mass of the corpuscles, and will vanish if either of these masses is zero. For consider what takes place when the electric force in the light-wave strikes the atom. Since the wave-length is large compared with an atom, the latter may be regarded as being in a uniform electric field; under this field the corpuscles will be displaced in one direction, the positive sphere in the opposite; and if the force persists long enough, this displacement will go on until the force exerted on the corpuscles in their displaced position by the positive electricity is equal and

* Communicated by the Author.

opposite to the force exerted on them by the electric force in the light-wave. The displacement of the corpuscles relatively to the sphere of positive electricity will polarize the atom, and the collection of polarized atoms will increase the specific inductive capacity, and therefore the refractive index of the medium. If the mass of either the positive electricity or of the corpuscles were zero, then, however short the time for which the electric force in the light-wave acted, the corpuscles and the positive electricity would adjust themselves in exactly the same way as if the electric force were continuous; so that the specific inductive capacity and the refractive index would be the same for short waves as for long, and there would be no dispersion. If, however, the masses of both the positive electricity and the corpuscles are finite, the relative displacement of the corpuscles and the positive electricity will depend upon the period of the electric force; and since the specific inductive capacity and refractive index depend upon this displacement, the refractive index of the medium will depend upon the period of the electric force, and there will be dispersion.

In the latter part of the paper the expression for the refractive index of a monatomic gas is investigated; and it is shown that if μ is the refractive index of such a gas for light of frequency p, then

$$\frac{\mu^2-1}{\mu^2+2}=\frac{\frac{4}{3}\pi\mathrm{NE}(\mathrm{M}e+m\mathrm{E})}{\frac{4}{3}\pi\rho(\mathrm{M}e+m\mathrm{E})-\mathrm{M}mp^2}, \quad . \quad . \quad . \quad (1)$$

where N is the number of atoms in unit volume of the gas, m the mass of a corpuscle, M the mass of the sphere of positive electrification, e the charge on a corpuscle, E the whole charge on the sphere of positive electrification, ρ the density of the electrification in this sphere; e, E, and ρ are expressed in electrostatic measure.

If the term in p^2 is small, equation (1) may be written

$$\frac{\mu^2-1}{\mu^2+2}=\frac{\mathrm{NE}}{\rho}\left(1+\frac{\mathrm{M}}{\mathrm{E}}\frac{m}{e}\,\frac{p^2}{\mathrm{M}+nm}\,\frac{3\mathrm{E}}{4\pi\rho}\right),$$

since $\mathrm{E}=ne$, where n is the number of corpuscles in the atom. If a is the radius of the sphere of positive electrification, $\mathrm{E}=\frac{4}{3}\pi\rho a^3$, *i. e.* $\frac{\mathrm{NE}}{\rho}=\mathrm{N}\frac{4}{3}\pi a^3=$ volume of the atoms per cubic centimetre of gas; this is the value of μ^2-1/μ^2+2 when $p=0$, *i. e.* for infinitely long waves. Writing P_0 for

this quantity, we see, if λ is the wave-length of the light,

$$\frac{\mu^2-1}{\mu^2+2}=\mathrm{P}_0+\mathrm{P}_0{}^2\frac{\mathrm{M}}{\mathrm{E}'}\frac{m}{e'}\frac{1}{\mathrm{N}(\mathrm{M}+nm)}\frac{3\pi}{\lambda^2}, \quad . \quad . \quad (2)$$

where E' and e' are the values of E and e in electromagnetic measure. This is the expression for the refractive index of a monatomic gas. I have not been able to find any determinations of the dispersion of such gases. Lord Rayleigh, however, found that the dispersion of helium was of the same *order* as that of diatomic gases. If the atoms in the molecules of a diatomic gas are not charged, the preceding expression will hold for the refractive index of such a gas; if, however, the atoms carry electrical charges, the theory subsequently given shows that this expression has to be modified. We know that as a matter of fact the atomic refraction of some elements, oxygen for example, depends upon the kind of compound in which the oxygen is found. This variation may be ascribed to the charges carried by the atoms in the molecules. The atomic refraction of hydrogen seems, however, to be constant; and I shall, in the absence of data for monatomic gases, apply the preceding formula to this gas.

From Ketteler's measurements of the refractive index of hydrogen for light of different wave-lengths, we find that for hydrogen at atmospheric pressure

$$\frac{\mu^2-1}{\mu^2+2}=\frac{1}{3}\left\{2{\cdot}8014\times10^{-4}+\frac{2\times10^{-14}}{\lambda^2}\right\}.$$

Comparing this with the equation

$$\frac{\mu^2-1}{\mu^2+2}=\mathrm{P}_0+\mathrm{P}_0{}^2\frac{\mathrm{M}}{\mathrm{E}'}\frac{m}{e'}\frac{1}{\mathrm{N}(\mathrm{M}+nm)}\frac{3\pi}{\lambda^2},$$

we find

$$\frac{\mathrm{M}}{\mathrm{E}'}\frac{m}{e'}\frac{1}{\mathrm{N}(\mathrm{M}+nm)}=6\times10^{-8};$$

but $\quad m/e'=1/1{\cdot}7\times10^7 \quad$ and $\quad \mathrm{N}e'={\cdot}8$;

hence

$$\frac{\mathrm{M}}{\mathrm{M}+nm}\frac{e'}{\mathrm{E}'}=1, \text{ approximately};$$

or, since $\mathrm{E}'=ne'$,

$$\frac{\mathrm{M}}{\mathrm{M}+nm}\frac{1}{n}=1, \text{ approximately}.$$

This result shows (1) that n cannot differ much from unity, and (2) that M, the mass of the carriers of positive electricity,

cannot be small compared with nm, the mass of the carriers of negative electricity. Hence we infer that n, the number of corpuscles in a hydrogen atom, is not much greater than unity. This result has been deduced from the consideration of the properties of a diatomic molecule; and if the atoms in the molecule were charged, the expression for $(\mu^2-1)/(\mu^2+2)$ would have to be modified; but since the dispersion of helium is by Lord Rayleigh's result comparable with that of hydrogen, we see, since the dispersion is proportional to $\frac{M}{M+nm}\frac{1}{n}$, that there cannot be a very large number of corpuscles in the helium atom; for if n were large, the dispersion of helium would be far too small.

2nd Method. Scattering of Röntgen Radiation by Gases.— It is shown in my 'Conduction of Electricity through Gases' that when Röntgen rays pass through a medium in which there are N corpuscles per cubic centimetre, the energy in the radiation scattered per cubic centimetre of the medium is $\frac{8\pi}{3}\frac{Ne^4}{m^2}E$, where E is the energy of the primary radiation passing through the cubic centimetre, e the charge, and m the mass of the corpuscle. Barkla has shown that in the case of gases the energy in the scattered radiation always bears, for the same gas, a constant ratio to the energy in the primary whatever be the nature of the rays, *i. e.* whether they are hard or soft; and secondly, that the scattered energy is proportional to the mass of the gas. The first of these results is a confirmation of the theory, as the ratio of the energy scattered to that in the primary rays is $\frac{8\pi}{3}N(e^4/m^2)$, and is independent of the nature of the rays; the second result shows that the number of corpuscles per cub. centim. is proportional to the mass of the gas: from this it follows that the number of corpuscles in an atom is proportional to the mass of the atom, *i. e.* to the atomic weight. Barkla measured the ratio of the energy in the scattered radiation to that in the primary in the case of air, and found that it was equal to $2{\cdot}4\times10^{-4}$. Thus, for air

$$\frac{8\pi}{3}\frac{Ne^4}{m^2}=2{\cdot}4\times10^{-4}.$$

Now $e/m=1{\cdot}7\times10^7$ and $e=1{\cdot}1\times10^{-20}$; hence

$$Ne=10.$$

But if n is the number of molecules per c. c.,

$$ne = \cdot 4;$$

hence $$N = 25n.$$

From this we deduce that there are 25 corpuscles in each molecule of air, and this indicates that the number of corpuscles in the atom is equal to the atomic weight; for the scattering by air is very nearly the same as that by nitrogen, and 25, the number of corpuscles in the molecule deduced from Barkla's experiment, is near to 28, the number in each molecule if the number in the atom were equal to the atomic weight.

3rd Method. Absorption of β Rays.—We regard the absorption of the β rays as due to the effect of the collisions between these rays and the corpuscles which they meet with in their path through the absorbing substance. If λ is the coefficient of absorption, it is shown in the latter part of the paper that

$$\lambda = \frac{4\pi N e^4}{m^2} \frac{V_0{}^4}{V^4} \log\left\{\frac{1}{2} \frac{aV^2}{V_0{}^2} \frac{m}{e^2} - 1\right\},$$

where N is the number of corpuscles per cubic centimetre, V the velocity of the β particles, V_0 the velocity of light, e the charge on a corpuscle in electromagnetic measure, m the mass of a corpuscle, and a a length comparable with the distance between the corpuscles in an atom.

If δ is the density of the absorbing substance, M the mass of an atom, n the number of corpuscles in the atom, we have $\frac{N}{n} M = \delta$; so that

$$\lambda = \delta \,.\, 4\pi \frac{e^2}{m^2} \frac{en}{M} \cdot e \frac{V_0{}^4}{V^4} \log\left(\frac{1}{2} \frac{aV^2}{V_0{}^2} \frac{m}{e^2} - 1\right).$$

Now λ/δ is approximately constant whatever be the nature of the absorbing substance; hence since the logarithmic term only varies slowly, we conclude that n must be proportional to M, *i. e.* that the number of corpuscles in an atom is proportional to the atomic weight. To find the number of corpuscles in an atom, let us apply the formula to the case of the β particles from uranium, for which, as Becquerel has shown, $V = 1\,6 \times 10^{10}$; and Rutherford finds that for copper and silver $\lambda/\delta = 7$. Putting $e/m = 1{\cdot}7 \times 10^{10}$, $e = 10^{-20}$, $V_0 = 3 \times 10^{10}$, we get

$$\frac{ne}{M} = \frac{1{\cdot}4 \times 10^4}{\log\left(\frac{mV^2}{V_0{}^2} \frac{a}{e^2} - 1\right)}.$$

The value of the logarithmic term is somewhat uncertain, involving as it does the indeterminate quantity a; it cannot, however, be large enough to alter the order of the term on the right-hand side. If M' is the mass of the hydrogen atom,

$$\frac{e}{M'}=10^4;$$

hence we have

$$n=\frac{M}{M'}\frac{1\cdot 4}{\log\left(\frac{mV^2}{V_0^2}\frac{a}{e^2}-1\right)};$$

i. e., n is of the same order as M/M' the atomic weight of the element.

Thus these three very different methods all lead to the result that the number of corpuscles in the atom of an element is of the same order as the atomic weight of the element; and from the first method we conclude that the mass of the carrier of unit positive charge is large compared with that of the carrier of unit negative charge. If we suppose the whole mass of an atom to be that of its charged parts, e/m for positive unit charge would be of the order 10^4.

An obvious argument against the number of corpuscles in the atom being as small as these results indicate, is that the number of lines showing the Zeeman effect, which must therefore be due to the vibrations of corpuscles, in the spectrum, say, of iron is very much greater than the atomic weight of iron. This objection would be conclusive if it could be shown that all these lines are due to the vibrations of corpuscles inside the normal atom of iron; but I submit that there is no evidence that this is the case. When an atom of an element is giving out its spectrum either in a flame or in an electric discharge, it is surrounded by a swarm of corpuscles; and combinations, not permanent indeed, but lasting sufficiently long for the emission of a large number of vibrations, might be expected to be formed. These systems would give out characteristic spectrum-lines; but these lines would be due, not to the vibrations of corpuscles inside the atom, but of corpuscles vibrating in the field of force outside the atom. Such lines would not be reversed by cold vapour, though they might be by very hot vapours, by the vapours in flames or in the neighbourhood of an electric discharge: the number of lines showing the Zeeman effect reversed by cold vapours is, however, very limited.

We shall now proceed to consider the theory of the method on which the preceding results are based.

Index of Refraction of a Collection of Atoms.

If an atom consisting of corpuscles dispersed through a sphere of uniform positive electrification is in the path of a wave of light, the electric force in the wave will displace the corpuscles in the atom; the motion of these charged corpuscles will produce a magnetic field in addition to that in the wave before it struck the corpuscle; the existence of this field will alter the velocity of propagation of the wave by an amount which we shall attempt to calculate.

Index of refraction of a monatomic gas whose atoms contain as much positive as negative electricity.—Consider an element of volume so small that throughout it the electric force in the wave may be regarded as constant. Throughout this volume the atoms will all be affected by the electric force in the same way.

If ξ_r, η_r, ζ_r are the displacements parallel to the axes of x, y, z of the rth corpuscle of an atom, x the displacement of the centre of the sphere of positive electrification, e the negative charge on a corpuscle, E the charge of positive electrification in the sphere, N the number of atoms per unit volume; then X′, Y′, Z′, the components of the electric force due to the displacement of the corpuscles, are given by the equations

$$\left.\begin{aligned} X' &= \tfrac{4}{3}\pi N(Ex - \Sigma e\xi_r) \\ Y' &= \tfrac{4}{3}\pi N(Ey - \Sigma e\eta_r) \\ Z' &= \tfrac{4}{3}\pi N(Ez - \Sigma e\zeta_r) \end{aligned}\right\}; \quad . \quad . \quad . \quad . \quad (1)$$

the summation is for all the corpuscles in one atom.

The equations of motion for the corpuscles and sphere of positive electrification are

$$M\frac{d^2x}{dt^2} = (X + X')E - \tfrac{4}{3}\pi\rho e\Sigma(x - \xi_r),$$

$$m\frac{d^2\Sigma\xi_r}{dt^2} = -(X + X')E + \tfrac{4}{3}\pi\rho e\Sigma(x - \xi_r),$$

where m is the mass of a corpuscle and M that of the sphere of positive electrification.

If all the quantities vary as $\epsilon^{\iota pt}$, we get from these equations

$$x = \frac{(X + X')Em}{\tfrac{4}{3}\pi\rho(Me + mE) - mMp^2},$$

$$\Sigma\xi_r = -\frac{(X + X')EM}{\tfrac{4}{3}\pi\rho(Me + mE) - mp^2}.$$

Therefore from equation (1) we have

$$X' = \frac{\frac{4}{3}\pi N(X+X')(mE^2+MEe)}{\frac{4}{3}\pi\rho(Me+mE)-mMp^2},$$

or

$$X' = \frac{PX}{1-P};$$

where

$$P = \frac{\frac{4}{3}\pi N(mE^2+MEe)}{\frac{4}{3}\pi\rho(Me+mE)-mMp^2}.$$

In consequence of the motion of the charged corpuscles, the current is no longer the polarization-current $\frac{K_0}{4\pi}\frac{dX}{dt}$, where K_0 is the specific inductive capacity of the æther; but this current *plus* the convection current $N\left(E\frac{dx}{dt}-e\Sigma\frac{d\xi_r}{dt}\right)$; thus u the total current parallel is given by the equation

$$u = \frac{K_0}{4\pi}\frac{dX}{dt} + \frac{3}{4\pi}\frac{dX'}{dt}.$$

If α, β, γ are the components of the magnetic force,

$$4\pi u = \frac{d\beta}{dz} - \frac{d\gamma}{dy},$$

and

$$\frac{d\beta}{dt} = \frac{dX}{dz} - \frac{dZ}{dx}, \quad \frac{d\gamma}{dt} = \frac{dY}{dx} - \frac{dX}{dy}.$$

Hence, since

$$\frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz} = 0,$$

we have

$$4\pi\frac{du}{dt} = \frac{d^2X}{dx^2} + \frac{d^2X}{dy^2} + \frac{d^2X}{dz^2};$$

or

$$K_0\frac{d^2X}{dt^2} + \frac{3d^2X_1}{dt^2} = \frac{d^2X}{dx^2} + \frac{d^2X}{dy^2} + \frac{d^2X}{dz^2},$$

$$K_0\frac{d^2X}{dt^2} + \frac{3P}{1-P}\frac{d^2X}{dt^2} = \frac{d^2X}{dx^2} + \frac{d^2X}{dy^2} + \frac{d^2X}{dz^2}.$$

Hence, if μ is the refractive index,

$$\mu^2 = 1 + \frac{3P}{1-P},$$

or

$$\frac{\mu^2-1}{\mu^2+2} = P = \frac{\frac{4}{3}\pi N(mE^2+mEe)}{\frac{4}{3}\pi\rho(Me+mE)-mMp^2}. \quad . \quad . \quad (2)$$

For very long waves, when the term mMp^2 may be neglected, we have

$$P = \frac{\frac{4}{3}\pi NE}{\frac{4}{3}\pi\rho};$$

or, since $E = \frac{4}{3}\pi\rho a^3$, where a is the radius of the sphere of positive electrification,

$$P = \frac{\mu^2-1}{\mu^2+2} = \frac{4}{3}\pi N a^3.$$

This is the value given by Mossotti's theory when the atoms are assumed to be perfectly conducting spheres of radius a.

Case of a diatomic molecule, the atoms carrying electrical charges equal in magnitude and opposite in sign.—In this case the refractivity P will contain a term due to the motion of the charged atoms relatively to each other under the electric field due to the light-wave. We can calculate this term as follows:—Let M_1, M_2 be the masses of the two atoms, E' the charge on the first atom, $-E'$ that on the other. Let x_1 be the displacement of the centre of the first atom, x_2 that of the other. The equations of motion will be of the form

$$M_1 \frac{d^2 x_1}{dt^2} = XE' - \phi(D)(x_1 - x_2),$$

$$M_2 \frac{d^2 x_2}{dt^2} = -XE' - \phi(D)(x_2 - x_1),$$

where $\phi(D)$ is a function of D, the distance between the atoms, the value of this function depending on the law of force. If the variables as before vary as $\epsilon^{\iota pt}$, we find

$$x_1 = \frac{M_2 E' X}{(M_1+M_2)\phi(D) - M_1 M_2 p^2},$$

$$x_2 = -\frac{M_1 E' X}{(M_1+M_2)\phi(D) - M_1 M_2 p^2}.$$

The contribution to P of the motion due to the coordinates x_1, x_2 is proportional to

$$N'\left(E'\frac{dx_1}{dt} - E_1'\frac{dx_2}{dt}\right)$$

or to

$$\frac{N' E_1{}^2 (M_1+M_2)}{(M_1+M_2)\phi(D) - M_1 M_2 p^2}, \quad . \quad . \quad . \quad . \quad (3)$$

where N' is the number of these diatomic molecules per unit volume. The consideration of this expression shows that

the part of the refractivity arising from the coupling of two atoms together may easily be comparable with the part due to the corpuscles within the atom. Thus, to take the case when the waves are so long that we may neglect the term in p^2, the contribution to the refractivity due to the coupling is

$$\frac{N'E_1^2}{\phi(D)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

If the force between the atoms changed as slowly as the force between the two charges E' and $-E'$ at a distance D, $\phi(D)$ would equal $2E'^2/D^3$, and (1) would become

$$\frac{N'D^3}{2}.$$

If we compare this with $2N'a^3$, the value due to the corpuscles inside the atom, a being the radius of the atom, we see that unless the force between the two atoms varies very rapidly with the distance, a considerable part of the refractivity may be due to the coupling between the atoms.

If ΔP_0 is the part of $(\mu^2-1)/(\mu^2+2)$ when λ is infinite due to the charges on the atoms in the diatomic molecule, we see from equation (3) that the part of $(\mu^2-1)/(\mu^2+2)$ due to these charges is approximately equal to

$$\Delta P_0+(\Delta P_0)^2\frac{M_1}{E_1'}\frac{M_2}{E_1'}\frac{1}{(M_1+M_2)}\frac{4\pi^2}{N\lambda^2}, \quad . \quad . \qquad (5)$$

where E_1' is the value of the charge in electromagnetic units.

In the case of a molecule consisting of two charged atoms, the charge on the negative atom will be due to the presence on the atom of extra corpuscles which can move freely about. Thus, if M_2 refers to the negative charge, M_2/E_1' will equal m/e_1, where m is the mass and e the charge on a corpuscle; for the positive atom M_1 will equal the mass of the atom, while E_1 will equal e_1 if the atoms are monovalent, $2e_1$ if they are divalent, and so on. Comparing the part of the coefficient of $1/\lambda^2$ which is due to the charge on the atoms, with that (given by equation (2)) due to the corpuscles inside the atom, we see that the factor M_2/E_1' in (5) is the same as m/e' in (2); while if there are many corpuscles in the atom, the factor $\frac{M_1}{E_1'}\frac{1}{M_1+M_2}$ will be much larger than $\frac{M}{E(M+nm)}$, for E_1' will only be a small multiple of e_1, while E is equal to ne_1, where n is the number of corpuscles in the atom. Thus, unless ΔP_0 is very small compared with P_0, the dis-

persion of the gas will depend more on the charges of the atoms in a diatomic gas than on the corpuscles inside the individual atoms.

The theory of the second method is given in my 'Conduction of Electricity through Gases.' We proceed to the consideration of the third method, which depends on the absorption by matter of rapidly moving corpuscles.

If we suppose that an atom consists of a number of corpuscles distributed through positive electrification, we can find an expression for the absorption experienced by the corpuscles when they pass through a collection of a large number of such atoms. The rapidly moving corpuscle will penetrate the atom, and will be deflected when it comes near an inter-atomic corpuscle by the repulsion between the corpuscles. This deflexion will produce an absorption of the cathode particles. If the corpuscle in the atom is held fixed by the forces acting upon it, the colliding corpuscle will, after the collision, have the same velocity as before, though the direction of its motion will be deflected. If the inter-atomic corpuscle A is not fixed, the colliding corpuscle B will communicate some energy to it and will itself go off with diminished energy. Without solving the very complicated problem which presents itself when we take into account the forces exerted on A by the other corpuscles, we can form some idea of the effects produced by the constraint introduced by such forces by following the effects produced by increasing the mass of A. The general effect of great constraint would be represented by supposing the mass of A to be very large, while absence of constraint would be represented by supposing the mass of A to be equal to that of B.

Let M_1, M_2 be the masses of the corpuscles A and B respectively. We shall suppose that the velocity of the colliding corpuscle is so great that in comparison the corpuscles in the atom may be regarded as at rest. Let V be the velocity of A before the collision, b the perpendicular let fall from A on V. If 2θ is the angle through which the direction of relative motion is deflected by the collision, we can easily show that

$$\sin^2\theta = \frac{1}{1+\frac{b^2V^4}{e^4}\left(\frac{M_1M_2}{M_1+M_2}\right)^2};$$

the force between two corpuscles separated by a distance r being assumed equal to e^2/r^2. Hence, if u, u' are the

velocities of B parallel to x before and after the collision,

$$u'-u=-\frac{M_1 u}{M_1+M_2}2\sin^2\theta+\frac{M_1}{M_1+M_2}\sin 2\theta\cos\phi\sqrt{V^2-u^2},$$

where ϕ is the angle between the plane containing b and V and that containing V and x. Averaging, the term containing $\cos\phi$ will disappear, and we have

$$u'-u=-\frac{2M_1 u}{M_1+M_2}\frac{1}{1+\frac{b^2V^4}{e^4}\left(\frac{M_1M_2}{M_1+M_2}\right)^2}.$$

If there are N of the interatomic corpuscles per unit volume, the number of collisions in which b is between b and $b+db$ made by a corpuscle B when it travels over a distance Δx is $N\Delta x \,.\, 2\pi b \,.\, db$. Hence, if U is the sum of the values of u for the B corpuscles per unit volume, and $\Delta(U)$ the change in U in the distance Δx,

$$\Delta(U)=-2UN\,.\,\Delta x\,.\,\frac{M_1}{M_1+M_2}\int_0^{b_1}\frac{2\pi b\,db}{1+\frac{b^2V^4}{e^2}\left(\frac{M_1M_2}{M_1+M_2}\right)^2}; \quad (6)$$

the upper limit being determined by the condition that B comes into collision with the A corpuscles one at a time, so that the shortest distance between B and the corpuscle with which it comes into collision must be small compared with a, the distance between two corpuscles. If r is the shortest distance between the A and B corpuscles, we can easily show that

$$1-\frac{b^2}{r^2}=\frac{2e^2}{V^2r}\frac{M_1+M_2}{M_1M_2}.$$

Putting $r=a$, we see that b' is of order

$$a\left(1-\frac{2e^2}{V^2a}\frac{M_1+M_2}{M_1M_2}\right)^{\frac{1}{2}}.$$

Integrating the expression on the right-hand side of equation (6), we get

$$\frac{dU}{dx}=-2U\frac{N\,.\,M_1}{M_1+M_2}\frac{e^4(M_1+M_2)^2}{V^4(M_1M_2)^2}\log\left(1+\frac{b_1{}^2V^4}{e^2}\left(\frac{M_1M_2}{M_1+M_2}\right)^2\right).$$

Since the logarithmic term only varies slowly, we may put for b_1 any quantity of the same order without greatly affecting the result, putting

$$b'=a\left(1-\frac{2V^2}{e^2a}\frac{M_1+M_2}{M_1M_2}\right)^{\frac{1}{2}}$$

$$\frac{d}{dx}(U)=-U\frac{4\pi Ne^4}{V^4}\frac{(M_1+M_2)}{M_1M_2{}^2}\log\left(\frac{aV^2}{e^2}\frac{M_1M_2}{M_1+M_2}-1\right).$$

Thus U, the number of corpuscles crossing unit area in unit time, varies as $\epsilon^{-\lambda x}$, where

$$\lambda = \frac{4\pi N e^4}{V^4} \frac{(M_1 + M_2)}{M_1 M_2^2} \log \left(\frac{a V^2}{e^2} \frac{M_1 M_2}{M_1 + M_2} - 1 \right);$$

λ is the coefficient of absorption. If $M_1 = M_2$, *i. e.* if the corpuscles are quite free to move in the atom,

$$\lambda = \frac{8\pi N e^4}{V^4 M_2^2} \log \left(\frac{1}{2} \frac{M_2 a V^2}{e^2} - 1 \right).$$

If M_1 is infinite, *i. e.* if the corpuscles are held fixed by the forces between them, we have

$$\lambda = \frac{4\pi N e^4}{V^4 M_2^2} \log \left(\frac{1}{2} \frac{M_2 a V^2}{e^2} - 1 \right),$$

the value used in Method 3.

We can get an approximate value of λ very simply by the following method. Consider a stream of corpuscles moving horizontally; the forward motion of any particle will be stopped by a collision in which its direction of motion is turned through an angle equal to or greater than a right angle; *i. e.* if θ is equal to or greater than $\frac{\pi}{4}$ or $\sin^2\theta > \frac{1}{2}$, $\sin^2\theta$ will be greater than 1/2 if

$$b^2 < \frac{e^4}{V^4} \left(\frac{M_1 + M_2}{M_1 M_2} \right)^2.$$

The number of collisions made by a corpuscle for which b is not greater than this value, as the corpuscle moves over a distance Δx, is

$$\frac{\pi N e^4}{V^4} \left(\frac{M_1 + M_2}{M_1 M_2} \right)^2 \Delta x.$$

Hence, if U is the number of corpuscles crossing unit area in unit time, we have, if we neglect the effect of collisions which do not result in a total stoppage of the particle,

$$\frac{dU}{dx} = -U \cdot \frac{N\pi e^4}{V^4} \left(\frac{M_1 + M_2}{M_1 M_2} \right)^2,$$

or, if λ is the coefficient of absorption,

$$\lambda = \frac{N\pi e^4}{V^4} \left(\frac{M_1 + M_2}{M_1 M_2} \right)^2.$$